COBEM 99
XV Congresso Brasileiro de Engenharia Mecânica
15th Brazilian Congress of Mechanical Engineering
22 - 26 de Novembro de 1999 / November 22 - 26, 1999 Águas de Lindóia, São Paulo,

EFFICACY EVALUATION OF VIBRATION ISOLATION SYSTEMS USING TWO METHODS: THE TRANSMISSIBILITY AND THE TRANSMITTED POWER

Alberto Coronado M.
Pontifícia Universidade Católica do Rio de Janeiro, Departamento de Engenharia Mecânica
Rua Marquês de São Vicente, 225 - 22453-900 - Rio de Janeiro - Brasil
Fernando A. Rochinha
Universidade Federal do Rio de Janeiro, Departamento de Engenharia Mecânica
C.P.: 68503, CEP 21945-970 - Rio de Janeiro - Brasil
Rubens Sampaio
Pontifícia Universidade Católica do Rio de Janeiro, Departamento de Engenharia Mecânica
Rua Marquês de São Vicente, 225 - 22453-900 - Rio de Janeiro - Brasil

Abstract. *Isolation systems are used in a large variety of applications to reduce the transmission of mechanical vibrations caused by or transmitted to equipment. In order to measure the efficacy of any isolation system there are essentially two methods, one based in the transmissibility and another in the transmitted power. We compare the two methods in a simple system composed of a flexible base (beam supported in six points, four of which fixed and the other two to be chosen so as to minimize the vibration of the bar), a flexible isolator and a rigid source. The excitation can be either a force or a moment. We show the ambiguity that arises due to the use of the classical transmissibility and its limitations, whereas for the power there is no ambiguity and the concept is easier to apply.*

Keywords: *Vibration isolation, Transmissibility, Transmitted power.*

1. INTRODUCTION

Isolation systems are used in a large variety of applications (automobiles, buildings, spatial structures like aircraft and satellites and, in rotating machines) so as to reduce the transmission of mechanical vibrations generated by or transmitted to equipment. In general the isolation problems can be classified in two types, as shown in "Fig. 1" .

In the first case (left-hand side) the environment is isolated from the vibrations generated by the machine, a typical example of which is the isolation of a car body from the vibrations generated by the engine. In the second case sensitive equipment is protected against the perturbations transmitted to the support structure, a common example which is the isolation of a bench with laser equipment.

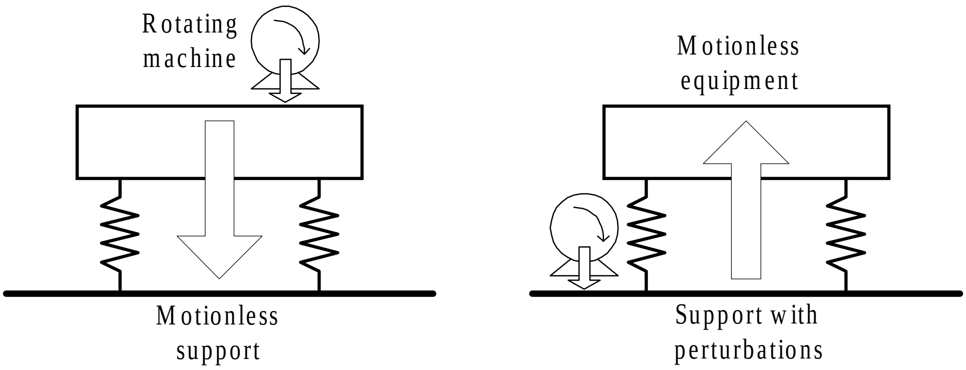

Figure 1: Vibration isolation types.

Isolation is obtained by inserting a mechanical component (isolator) which acts as a link between the sub-system that contains the perturbation and the one to be isolated.

A measure extensively used to evaluate the effectiveness of an isolation system is the classical transmissibility, which is in general described in the frequency domain. Depending on the use, this quantity can be expressed as the relation between the transmitted force to the base and the excitation force, or as the relation between the motion amplitude (displacement or velocity) of the protected equipment and the motion amplitude of the support. Hybrid formulations are also possible, e.g., the relation between the force and the displacement (or velocity) and vice-versa. A exposition of its use in simple systems of one degree of freedom is shown in (Inman, 1996), and for systems with several degrees of freedom in (Ponslet & Eldred, 1996).

Another measure is the transmitted power, which has been more and more studied and used, and is more appropriate to be minimized because it carries the displacements (or velocities) and the forces in a single scalar quantity.

In recent years it has been recognized the importance in the vibration transmission of the rotational degrees of freedom, in addition to the translational ones, especially in the high frequency range (Sanderson, 1996). In conventional studies of isolation systems, when the source and/or the base are not rigid (Snowdon, 1975), only translational movements normal to the support surface of the isolator were considered. But it is well known that the vibratory power exciting a beam or a plate due to the moments is more severe in the high frequency range (caused by the propagation of flexural waves) than the power related to the forces.

The unifying concept of vibratory power has been up to now the only way to overcome the hard problem of incompatible units which arises when it is necessary to compare the effects of a multi-directional excitation in a isolation system.

Another problem with the use of the classical transmissibility arises when an isolation system has several degrees of freedom and the isolated equipment has many rigid body modes. In this case the evaluation of the isolation quality considering all degrees of freedom is a difficult problem. In order to solve this problem there are some studies that use variations of the transmissibility concept, but they do not evaluate the effect of the moments (Ponslet & Eldred, 1996).

2. ISOLATION MODEL

2.1. Basic concepts and definitions

Considering "Fig. 3", it will be presented the expressions used in this work: the mobility (source and base) and impedance (isolators) matrix formulations.

All the forces and displacements have 3 components ($q_s = [f_y \;\; f_z \;\; t_x]^T$, $q_b = [f'_y \;\; f'_z \;\; t'_x]^T$, $f_s = [n_y \;\; n_z \;\; t_x]^T$, $v_s = [v \;\; w \;\; \theta_x]^T$, etc.), but we have adopted a compact notation for simplicity. Because each component is a quantity that varies harmonically it can be represented in the following way: $f_y = F_y e^{i\omega t}$, $f_z = F_z e^{i\omega t}$, $t_x = T_x e^{i\omega t}$, etc.

The expressions that relate the velocities ($\dot{v}$) with the forces (f) in the source and the base can be expressed as (Gardonio et al., 1997; Coronado, 1999):

$$\dot{v}_s = M_{s1}.f_s + M_{s2}.q_s \tag{1}$$

$$\dot{v}_b = M_{b1}.f_b + M_{b2}.q_b \tag{2}$$

Herein it will be used the following notation: the mobilities (M) with subscript 1 correspond to the internal forces and those with subscript 2 to the external forces.

In the isolator, the forces and the velocities in its ends will be related by the following form:

$$f_i = Z_i.\dot{v}_i \Rightarrow \dot{v}_i = Z_i^{-1}.f_i \tag{3}$$

Now, the general expressions that relate the external forces (q_s, q_b) with the internal velocities and forces ($\dot{v}_s$, $\dot{v}_b$, f_s, f_b) in the system will be derived.

Introducing the quantities:

$$M_{sb1} = \begin{bmatrix} M_{s1} & 0 \\ 0 & M_{b1} \end{bmatrix}, \quad M_{sb2} = \begin{bmatrix} M_{s2} & 0 \\ 0 & M_{b2} \end{bmatrix} \tag{4}$$

$$q_{sb} = \begin{bmatrix} q_s \\ q_b \end{bmatrix}, \quad \dot{v}_{sb} = \begin{bmatrix} \dot{v}_s \\ \dot{v}_b \end{bmatrix}, \quad f_{sb} = \begin{bmatrix} f_s \\ f_b \end{bmatrix} \tag{5}$$

The "Eq. (1) and (2)" can be written as:

$$\dot{v}_{sb} = M_{sb1}.f_{sb} + M_{sb2}.q_{sb} \tag{6}$$

Using the principle of action and reaction for the forces and enforcing the continuity for the velocities, it follows that:

$$f_i = -T.f_{sb} \Rightarrow f_{sb} = -T^{-1}.f_i \tag{7}$$

$$\dot{v}_i = T.\dot{v}_{sb} \Rightarrow \dot{v}_{sb} = T^{-1}.\dot{v}_i \tag{8}$$

Where T is a transformation matrix that relates the displacements, velocities or forces in the source (or base) with those in the isolator extremities.

Using the above expressions, it can be found that:

$$\dot{v}_{sb} = M_{\dot{v}}.q_{sb} \tag{9}$$

$$f_{sb} = M_f.q_{sb} \tag{10}$$

Where:

$$M_{\dot{v}} = (I + M_{sb1}.T^{-1}.Z_i.T)^{-1}M_{sb2} \tag{11}$$

$$M_f = -(T^{-1}.Z_i^{-1}.T + M_{sb1})^{-1}M_{sb2} \tag{12}$$

2.2. Power calculation

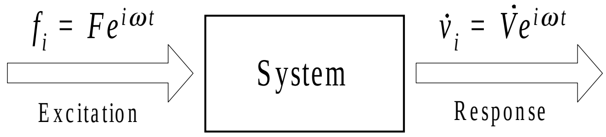

Figure 2: Power calculation.

The mean power in a period will be:

$$P = \frac{1}{T}\int_0^T f_i\dot{v}_i\, dt = \frac{1}{T}\int_0^T Re[Fe^{i\omega t}]Re[\dot{V}e^{i\omega t}]\, dt \tag{13}$$

Then the power can be calculated as:

$$P = \frac{1}{2}Re[F^*\dot{V}] = \frac{1}{2}Re[F\dot{V}^*] \tag{14}$$

3. Isolation model with a flexible base with 6 supports and a flexible isolator

The isolation model which is used to compare the transmissibility and the transmitted power is shown in “Fig. 3”. It is composed of a rigid source under a vertical force or a moment excitation (the model allow us to use a multi-directional excitation: a vertical force, a horizontal force and a moment), so as to permit the comparison with the transmissibility because it is limited to simple excitations. The isolator was modeled analytically as a continuum (it has infinite resonances). The base was also modeled as a continuum and its mobility expressions (since they are more complex) were calculated by the Finite Element Method using a substructuring technique.

The aim is to position supports 2 and 5 of the base so as to minimize its vibration. Support 2 is to be positioned between 1 and 3, and similarly 5 between 4 and 6.

In “Fig. 4” it is shown the input, intermediate and transmitted powers, when the excitation is a vertical force. In this case supports 2 and 5 were located close to 1 and 6,

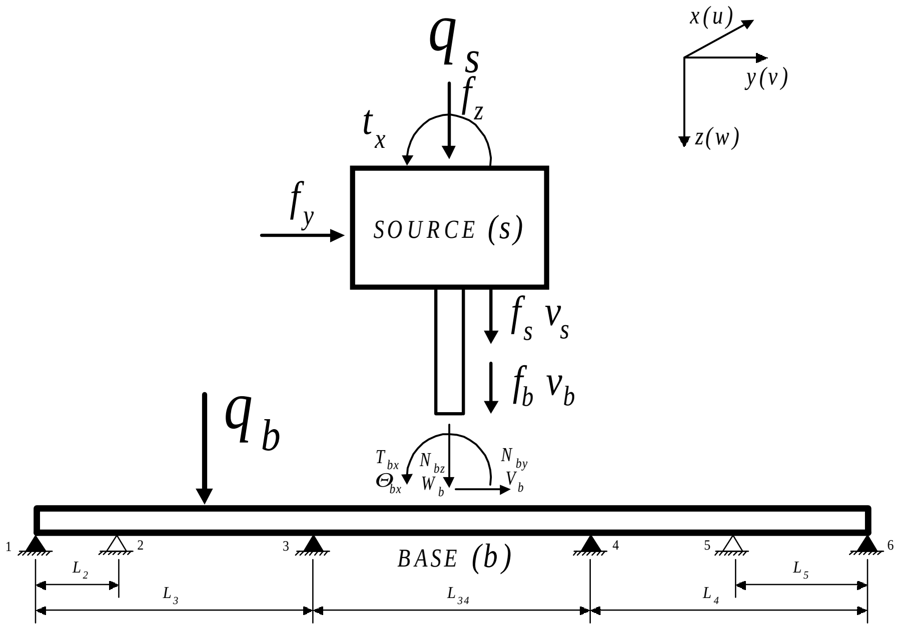

Figure 3: Isolation model

respectively ($L_2 = L_5 = 0,01m$). It can be noted that there are a resonance of the source close to 10 Hz, and two flexible modes of the isolator on the 100 - 1000 Hz frequency range.

In "Fig. 5" it is used the same excitation to compare the power transmitted to the base when supports 2 and 5 are close to 1 and 6 ($L_2 = L_5 = 0,01m$), and when they are close to 3 and 4 ($L_2 = L_5 = 0,29m$).

In "Fig. 6 and 7" it is shown how the power transmitted to the base varies with the location of supports 2 and 5. It can be seen that when they are in a region close to supports 3 and 4, respectively, the power is minimized for the two excitation types we analized, a vertical force $F_z = 1$ and a moment $T_x = 1$.

"Figure 8" shows the transmissibility variation, which is defined in this case as the relation between the vertical velocity of the base (at the contact point with the isolator), and the excitation force (vertical unitary). According to this figure, the region located close to supports 3 and 4 minimize this transmissibility, which agrees with "Fig. 6" of the transmitted power.

In "Fig. 9" it is shown the transmissibility variation, now defined as the relation between the vertical force on the base (at the contact point with the isolator) and the excitation force (vertical unitary). In this case, the region that minimizes this function is close to supports 1 and 6 respectively, which disagrees with "Fig. 6 and 8".

"Figures 10 and 11" show the transmissibility under another excitation, a unitary moment. In both cases, the transmissibilities are calculated with the vertical components of the velocities or forces, since it is well known that they are easier to be measured than the rotational components. In both figures it is shown that there are 13 points that minimize these transmissibilities and not only one.

In "Fig. 12 and 13" are shown transmissibilities under the same excitation (a unitary moment), but in this case the rotational component of the velocity and the moment in the base were considered. The region that minimizes the moment is completely opposed to that of "Fig. 7 and 12".

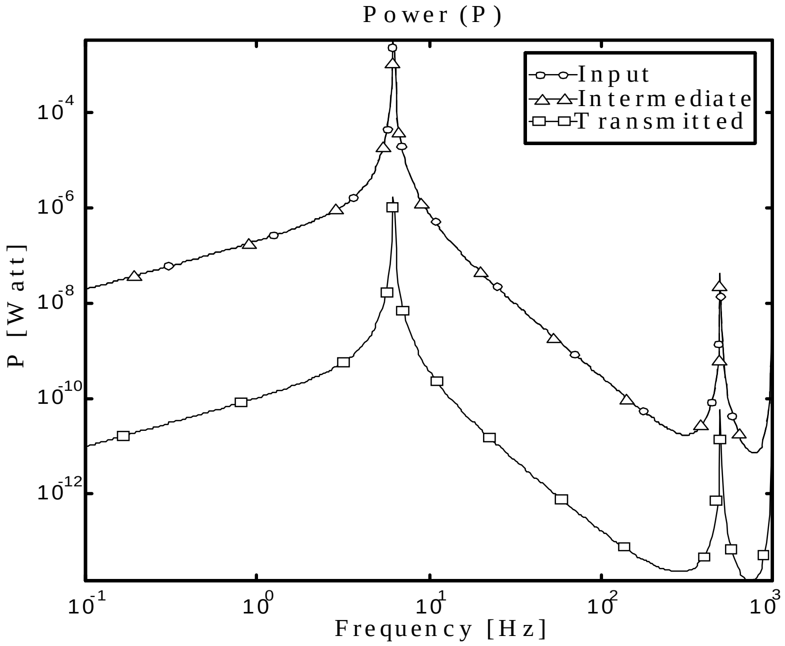

Figure 4: Transmitted power ($L_2 = L_5 = 0.01$), excitation: $F_y = 0, F_z = 1, T_x = 0$

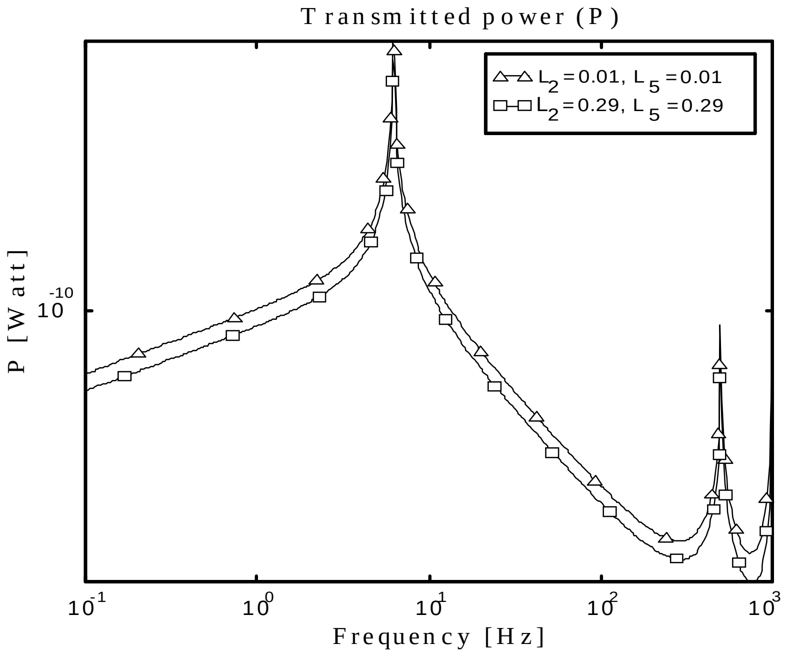

Figure 5: Variation of the transmitted power to the base, $F_y = 0, F_z = 1, T_x = 0$

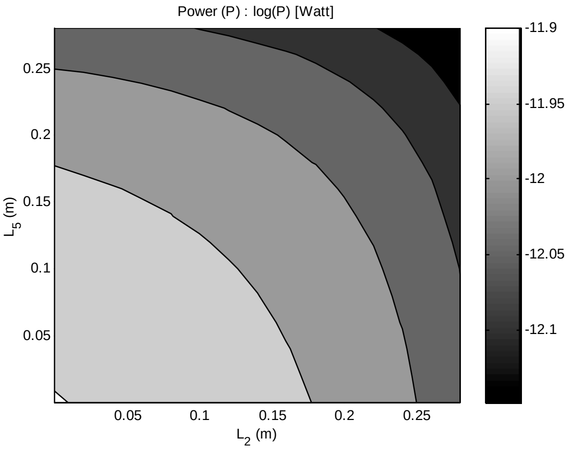

Figure 6: Transmitted power, $F_y = 0, F_z = 1, T_x = 0$ at 50 Hz

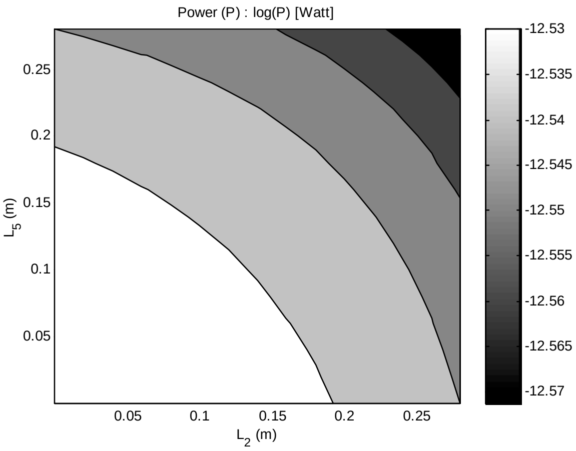

Figure 7: Transmitted Power, $F_y = 0, F_z = 0, T_x = 1$ at 50 Hz

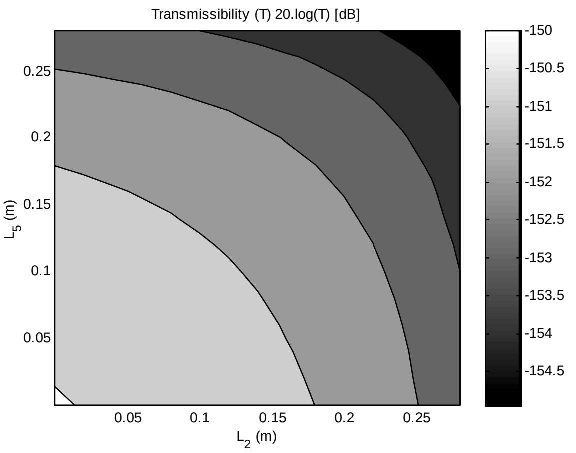

Figure 8: Transmissibility $\frac{\dot{W}_b}{F_z}$, $F_y = 0, F_z = 1, T_x = 0$ at 50 Hz

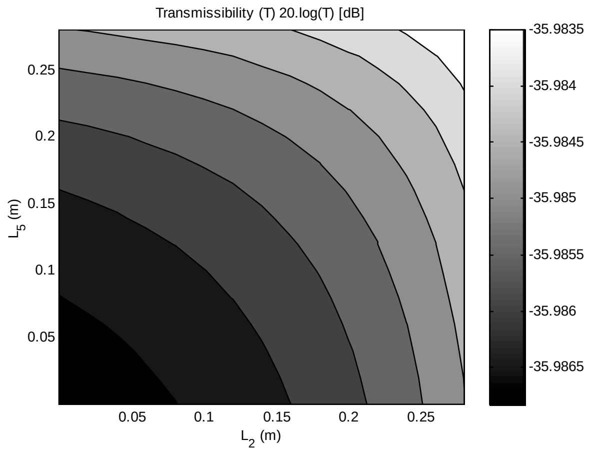

Figure 9: Transmissibility $\frac{N_{bz}}{F_z}$, $F_y = 0, F_z = 1, T_x = 0$ at 50 Hz

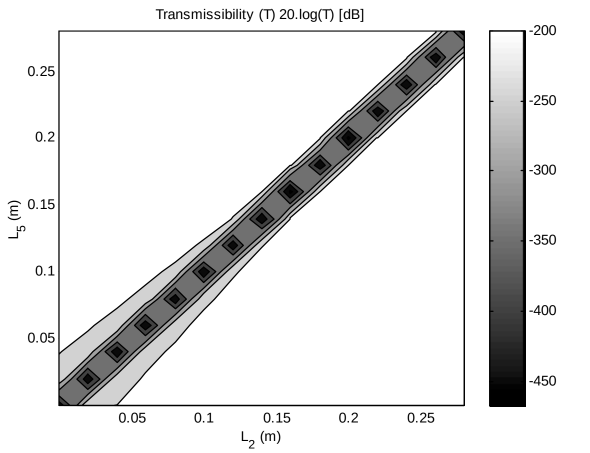

Figure 10: Transmissibility $\frac{\dot{W}_b}{T_x}$, $F_y = 0, F_z = 0, T_x = 1$ at 50 Hz

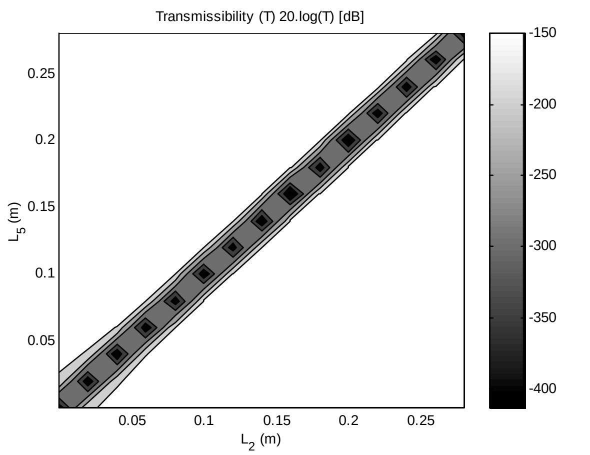

Figure 11: Transmissibility $\frac{N_{bz}}{T_x}$, $F_y = 0, F_z = 0, T_x = 1$ at 50 Hz

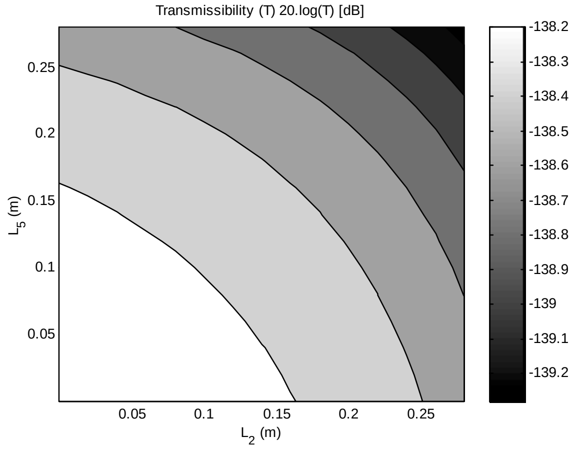

Figure 12: Transmissibility $\frac{\dot{\Theta}_{bx}}{T_x}$, $F_y = 0, F_z = 0, T_x = 1$ at 50 Hz

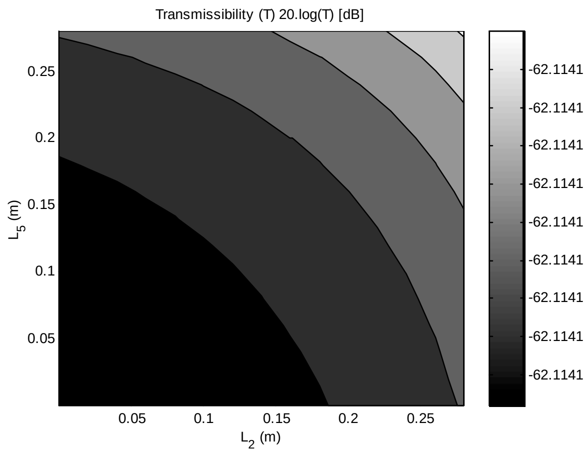

Figure 13: Transmissibility $\frac{T_{bx}}{T_x}$, $F_y = 0, F_z = 0, T_x = 1$ at 50 Hz

4. CONCLUSIONS

The classical transmissibility and the transmitted power, the most used methods to evaluate the effectiveness of an isolation system, were compared. The transmissibility (which is older and the more used) is inconsistent in its results when is defined in function of the velocities or forces. Moreover it is not easy to formulate a simple definition when the excitation and/or the response at the same point has more than one component.

In this context the power acts as a unified criterion since it allows us to evaluate systems under multi-directional excitations, with several isolators and components that present flexible modes. Moreover it incorporates in a single quantity the effects of the forces and velocities eliminating by this way any ambiguity.

REFERENCES

Coronado, M. A., 1999, Energy propagation in vibration isolation systems (in Portuguese), M.Sc. Thesis, Pontifícia Universidade Católica do Rio de Janeiro, Rio de Janeiro, Brasil, http://www.mec.puc-rio.br/prof/rsampaio/rsampaio.html

Gardonio, P., Elliot S. J. and Pinnington R. J., 1997, Active isolation of structural vibration on a multiple-degree-of-freedom system, Part I: The dynamics of the system, Journal of Sound and Vibration, 207(1),61-93.

Inman, D. J., 1996, Engineering Vibration, Prentice-Hall Inc.

Ponslet E.R. and Eldred, M. S., 1996, Discrete optimization of isolator locations for vibration isolation systems: An analytical and experimental investigation, Report SAND96-1169, May 17, Structural Dynamics Department, Sandia National Laboratories.

Sanderson, M. A., 1996, Vibration isolation: Moments and rotations included, Journal of Sound and Vibration, 198(2), 191-197.

Snowdon, J. C., 1975, Compound mounting systems that incorporate dynamic vibration absorbers, Journal of Engineering for Industry, ASME, November, 1204-1211.